

# Modal Analysis of Quill Shaft in Turbo-generator Unit Based on SolidWorks

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## ABSTRACT

The main goal of this work is to carry out a numerical modal analysis of a Quill shaft of turbo-generator unitaffiliate to Mosul gas turbine station, using a trail version of popular finite element analysis software SolidWorks. The main function of Quill shaft is to protect the turbo-generator unit against overloads due to electrical network faults. The high flexibility of this shaft makes it capable of absorbing high displacements of resonance phenomena. This analysis is essentially needed to study the effect of transient loads applied to Quill shaft of turbo-generator unit under severe loading conditions such as electric network disturbances. The first five values of critical frequencies and mode shapes of axial, bending, and torsional vibrations were studied and analyzed. Each mode has been isolated separately by applying a special type of boundary conditions (restraints) available in program. The three types of natural frequencies have been found and reported. It was observed that the fundamental values of each three types of natural frequencies are relatively high and out of the range of Quill shaft operating speed. Finally, it has been concluded from all analyses that Quill shaft under consideration is safe from the stand point of modal analysis. The results show that the Quill shaft is not running at any of each three types critical speeds. Therefore, the resonance phenomenon for all three types of vibrations can not be happened no matter how high the amount of transient load applied.

## Keywords:

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SolidWorks; Finite element method; Turbo-generator unit; Natural frequency; Modal analysis; Quill shaft; Simulation.

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## **1. INTRODUCTION**

S. M. J. Ali, Z. Y. Mohammad and F. Q. Yahya [1] developed a model, Fig.1, for actual turbogenerator unit of Mosul gas turbine station and governing equations of this model has been formulated to estimate their respective parameters. The turbo-generator unit shown in Fig.1, is exposed to high axial, bending, and oscillations torsional due to high disturbances induced in the electrical grid during the connection of the generator unit to that grid. The main reasons of these faults are: the shortcircuit, open-circuit, and sudden load changes. Inaddition, the outdoor parts of the electrical

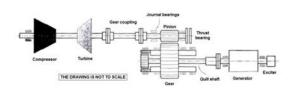


Fig.1 Turbo-generator unit

system (overhead lines, electrical towers, circuit breakers,....etc)experienced to different types of severe atmospheric conditions like ice, winds, moisture, ....etc, may play an important role in causing these types of faults. Qing He and Dongmei Du [2] showed that the coupling interaction between electrical network faults and torsional vibrations makesshaft of turbine-generator unitvibrate. Therefore, alternating torsional stresses due toresonate torsional vibrations. Also, when any type of faults occurs in the grid, the electromagnetic torque between stator and rotor changes rapidly. As a result, torsional vibrations of shaft occur, alternate shear stresses due to these vibrations decrease the shaft life, and may be break the shaft.

For this reason, it becomes important to compute and analyze the natural or resonatefrequencies and their corresponding mode shapes of all three types of vibrations(axial, bending, and torsional) agitated by the faults of electrical network, in order to obviate and stop disastrous accidents.

The present work is related to Mosul gas turbine station which contains multi turbo-generator units for producing electrical power. The most important and critical part of the turbo-generator unit is the Quill shaft. The purpose of Quill shaft is to connect the gearbox reducer with

the generator as shown in Fig.2. Its main function is to work as a mechanical fuse for the unit to protect it against overloads and electrical faults and disturbances. The high axial, bending, and torsional flexibility owned by Quill shaft material make it deformed or brake before any other part of the unit because of its high capacity to absorb the induced large axial, bending, and torsional displacements.

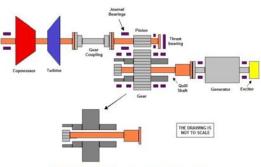


Fig.2 Quill shaft configuration

## **Objectives of the present work**

The main goal of the present work is to find the values of resonate natural frequencies of the Quill shaft for three conditions: axial, bending and torsional and its corresponding mode shapes, by using numerical method (Finite element method) and compare them with the forced frequency (actual operating speed of the shaft). If one or more of these natural frequencies is located within the value of operated speed of the shaft, then, it is said that the shaft is in resonance condition, the case that should be avoided in the initial or early design stages. However, in this case, a redesign or improved design of Quill shaft should be introduced. This step may be carried out in the future work.

## 2. MODAL ANALYSISTHEORY

## 2.1 Basic theory of modal analysis

Yichang Liu, ChunlinTian, Tao Liu, and Fanwu Kong [3] defined the modal analysis as a numerical technique to calculate the characteristics of structure. The vibration characteristics of any structure including critical frequency and mode shape.

Li Cheng and Hongguang Wang [4] stated that the critical frequency and mode shape considered the two significantfactors on the dynamic properties of any mechanical structure, which are played an important role in the design of the structural system for vibration application. Both of them depend on the mass and stiffness properties of the structure. In addition, the damping has loweffect on the ritical frequency and mode shape, therefore, the damping coefficient can be cancelled in the modal analysis. For a linear system of free vibrations, the differential equations be summarized in a matrix form[5].

## $\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{0}$ ....(1)

Where

**M** is themass matrix (n x n)

**K** is the stiffness matrix (n x n)

**X** is the n-dimensional column vector of generalized coordinates.

The general form of the differential equations for a free vibrations of a linear undampedn-degree of freedom system is

 $\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{0}....(2)$ 



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The solution of equation (2) can be given in the form

Where  $\omega$  is the vibration frequency and **X** is an n-dimensional vector of mode shape.

The values of  $\omega$  such that equation (3) is a solution of equation (2) are called the *natural frequencies*. Each natural frequency has at least one corresponding mode shape.

Substitution of Equation (3) into Equation (2) leads to

 $(-\omega^2 \mathbf{M} \mathbf{X} + \mathbf{K} \mathbf{X}) e^{i\omega t} = 0 \quad \dots \qquad (4)$ 

Since  $e^{i\omega t} \neq 0$ , for each real value of t,

 $-\omega^2 \mathbf{M} \mathbf{X} + \mathbf{K} \mathbf{X} = \mathbf{0}.....(5)$ 

The mass matrix Mis nonsingular, and thus  $M^{-1}$  exists. Multiplying equation (5) by  $M^{-1}$  and rearranging gives

Where  $\mathbf{I}$  is the identity matrix (n x n).

Equation (6) is the matrix form of a systemof nsimultaneous linear algebraic equations. Applying Cramer's rule to obtain the solution for the jth component of X, Xj, as

Thus the solution  $(\mathbf{X} = \mathbf{0})$  is obtained unless

 $\det |\mathbf{M}^{-1}\mathbf{K} - \omega^2 \mathbf{I}| = 0 \dots (8)$ 

Hence,  $\omega^2$  must be an eigenvalue of  $\mathbf{M}^{-1}\mathbf{K}$ . The mode shape is the corresponding eigen vector. Natural frequencies of multi-degree of freedom systems are computed as the quadratic roots of the eigenvalues of  $\mathbf{M}^{-1}\mathbf{K}$ , i.e.,

$$\omega_n = \sqrt{\frac{\kappa}{M}} \qquad (9)$$

Similarly, the equation of the angular motioncan be obtained by the same way [6]:

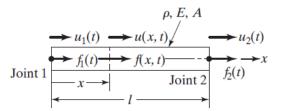
 $\mathbf{J}\ddot{\mathbf{\Theta}} + \mathbf{K}_{\mathbf{t}}\mathbf{\Theta} = \mathbf{0}....(10)$ 

which can be seen to be comparable to equation (2) if **J** and **K**<sub>t</sub> are the n x npolar mass moment of inertia and torsional stiffness matrices respectively, and  $\theta$  is the n-dimensional column vector of generalized coordinates. Thus the natural circular frequency of the torsional system is

# 2.2Steps of the FEMapplying to problems of vibration

The FEM is a numerical technique that can be used for an accurate butapproximate solution of many complex mechanical and structural vibration problems[6, 7].

In this method, theactual structure is replaced by several pieces or elements, each of which is assumed tobehave as a continuous structural member called a finite element. The elements areassumed to be interconnected at joints or nodes. The mass and stiffness matrices and force vectors needed for the finite element analysis can be derived for the basic onedimensional elements such as a bar in axial motion, a rod in torsional motion, and a beamin bending motion. The boundary conditions should be incorporated to the conditions to the assembled system matrices and equations.For clarification, an example was presented here of uniform bar considered as onedimensional element, the two end points form the joints or nodesas shown in Figure below[8].



The element is subjected to axial  $loadsf_i(t)$  and  $f_2(t)$ , the axial displacement *u* within the element is linear in *x*. The joint displacements  $u_1(t)$  and  $u_2(t)$  are unknowns. After several steps of derivation, the mass and stiffness matrices along with force vector can be identified as:

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And the force vector as:

$$\vec{f} = \begin{cases} f_1(t) \\ f_2(t) \end{cases} \tag{14}$$

Where  $\rho$ , E are density and modulus of elasticity of the bar material respectively, A is the crosssectional area, and l is bar length.

Most softwares using finite element techniques which included frequency analysis gives the results of all types of natural frequencies (axial, bending, and torsional) in a confusing or mixed way and not separately. This makes the sorting or isolating process of each type of natural frequencies is difficult, tedious, and consumes more time.

To facilitate this work and reduce the effort as well as the execution time of program running, a control on the selection of the type of constraints or fixtures used in the model is applied so that each of three types of natural frequencies can be found individually. Figs. 3, 4, and 5 show the types of fixtures (boundary conditions) used for axial, bending, and torsional frequencies analysis in SolidWorks.

#### **3. FINITE ELEMENT METHODOLOGY**

#### **3.1. Finiteelementmethod**

FEM simulates a solid part by splitting the itsgeometry into a number of standard shapeelements, applying loads and restraints, assign material, then computing variables of interest like deflection, stresses, .....etc. The behavior of each element is depicted by a set of equations. Just as the set of elements would be joined individuals elements are joined into a set of equations that depict the behaviors of the whole structure. Any continuous body has finite degree of freedom and its not possible to solve in this format. With the help of discretization, i.e. nodes and elements, FEM reduces degree of freedom from infinite to finite.All computations are made at finite number of points known a particular shape such as quadrilateral or triangular ....etc, is known element. To obtain the value of variable say displacement between the calculation points, interpolation function is used. The solution of any engineering problem is carried out by three methods: analytical method or hand calculations, numerical method, and experimental method or physical testing. FEM is classified under numerical method denomination. The program takes the elements which havebeen defined,puts the equationsfor each unknown value, and collects them together as equation of matrix form. Finally,solve allthese for the unknown parameters

Table 1: Material properties of Quill shaft

|   | Alloy Steel           |                        |
|---|-----------------------|------------------------|
| 1 | Yield Strength (Sy)   | 620.422 MPa            |
| 2 | Tensile Strength (Su) | 723.825 MPa            |
| 3 | Elastic Modulus (E)   | 210 GPa                |
| 4 | Shear Modulus (G)     | 79 GPa                 |
| 5 | Poisson's Ratio (v)   | 0.28                   |
| 6 | Density (p)           | 7700 Kg/m <sup>3</sup> |

[9].

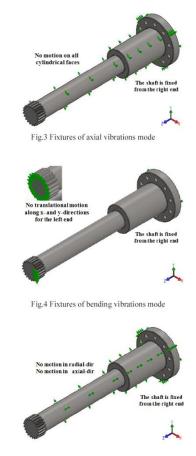


Fig.5 Fixtures of torsional vibrations mode

3.2 Finite element modeling and simulation



To construct the Quill shaft model using FEM method, a trial version of SolidWorks 2011 has been used. The following steps give the modeling and simulation for the modal frequency analysis carried out by SolidWorks:

#### 1. Model Building

Building a solid (3D) model of the Quill shaft using the actual dimensions extracted from Mosul gas turbine station. The solid (CAD) model and detailed dimensions of the shaft are shown in Figs.6 and 7 respectively.



Fig. 6 Solid model of Quill shaft

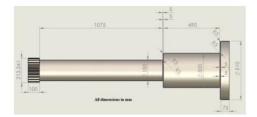


Fig. 7 Detailed drawing of Quill shaft



Fig.8 Mesh model of Quill shaft

#### 2. Material Selection

Assign the properties of the actual material fromwhich the Quill-shaft is manufactured. These properties are given in Table 1.

#### 3. Fixtures (boundary conditions) definition

Refer to Figs.3, 4, and 5 again here, show the three types of fixtures used in the present modal analysis to isolate axial, bending, and torsional frequencies of Quill Shaft. A constraint means exactly zero displacement in the direction required.

#### 4. Model Meshing

Results accuracy and speed of calculations are influenced directly by the type of mesh or grid used. Therefore, meshing is considered as the critical step in finite element analysis. Meshing of a geometric object represents a set of finite elements used for computational analysis or modeling. As the number of elements increased, the accuracy of results increased and the speed of calculations decreased. As a compromised solution, medium mesh rather than coarse or fine mesh has been used to obtain the accepted results, considering the computer operation ability. The type of element is solid have a minimum size of 17.32 mm and a maximum size of 86.632 mm has beenadopted. The mesh model of Quill shaft is shown in Fig. 8.

#### **5**.Solution Form Selection

FFEPlus is selected as the solution form as available in SolidWorks.

#### 6. Natural Frequency Determination

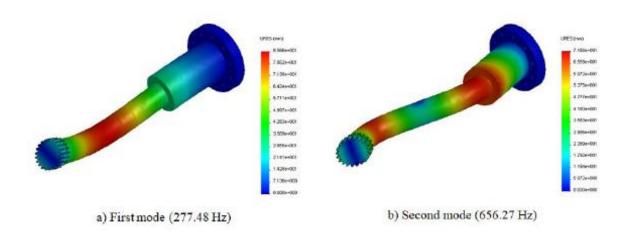
The first 5<sup>th</sup> natural frequencies has been calculated for each type (axial, bending, and torsion).The 1<sup>st</sup> natural frequency is always considered the fundamental or lower value which represents the closest value to the operated speed of the Quill shaft.

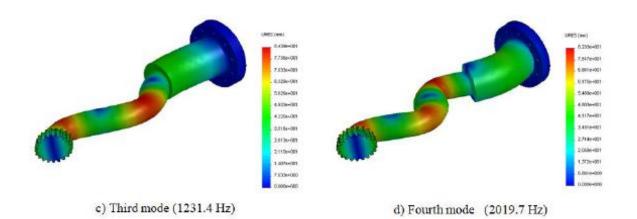
#### **7.** Final Results

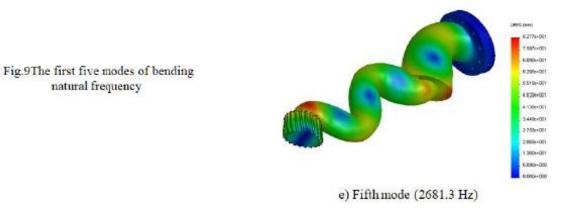
The eventual results of analysis can be acquired by running the program including critical speeds and inherent mode shapes for three types of vibrations.

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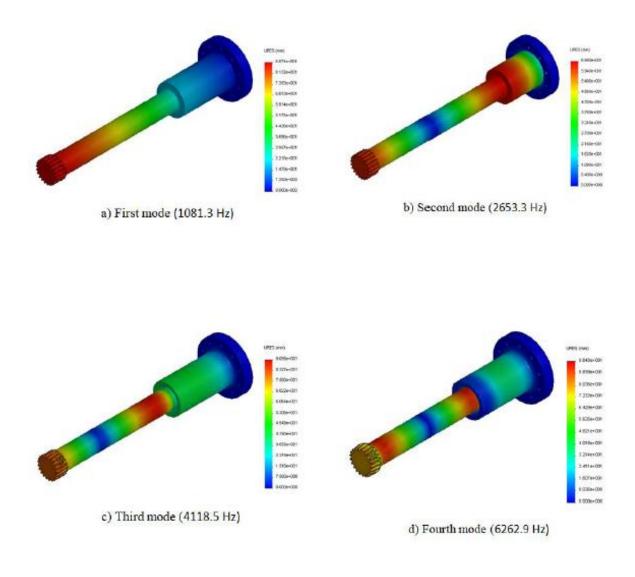


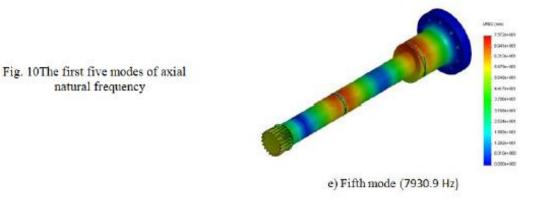


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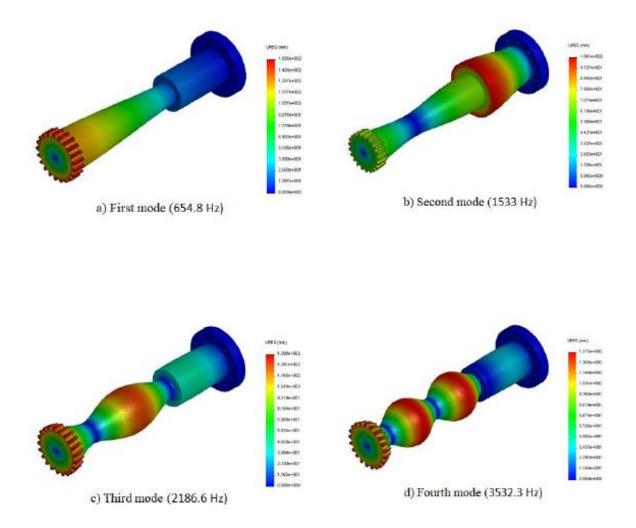


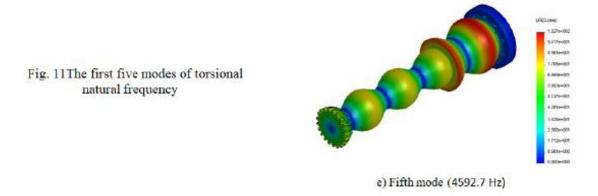


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#### 4. RESULTS

The vibration characteristics of Quill shaft in turbo-generator unit was obtained with modal analysis. The first five values of mode shapes of Quill shaft was found with their bending, axial and torsional natural frequencies and the deformation distribution. Each type of natural frequency has been separated in order to characterize each mode accurately and obviously. Table2 show thebending,axial and torsional natural frequencies of the first five modes of theQuill shaft, and the inherent mode shapes is displayed in Figs. 9, 10, and 11 respectively.

The results show that the values of natural frequency seems to be increasing from first modeshape to last mode shape for all three types as shown in Fig.12 in terms of cycle/sec (Hz), and Fig.13 in terms of revolution/min (rpm). Also, it is observed that the valuesof resonant natural frequencies are out of the rotating speed of Quill shaft in each type. The interval betweeneachnatural frequency mode is comparatively high for all three types as shown inTable2.

From the mode shapes shown in Figs. 9, 10, and 11, along with Table 3and Fig.14for maximum values of resultant displacements, it can be seen that the maximum amplitudes of shaft deformation or displacement occur at all modes for torsional natural frequencies, and the minimum amplitudes occur at second mode for axial natural frequency.

Also, it can be observed from Figs.9,10,and 11, or table 2, that the minimumvalue of resonant natural frequency which represents the nearest value to Quill shaft operating speed (3000 rpm=60 Hz) is the first mode of bending frequency (16648.8 rpm=85.66 Hz).

All types of natural frequencies (bending, axial, and torsional) are relatively high (out of the range of rotating speed of shaft, six times minimum for bending, 13.1 times medium for torsional, and 21.63 times high for axial) considered as safety margins comparing withfundamental natural frequency as reported in Tables 4, 5, and 6

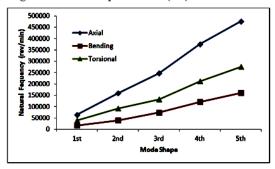
## **5. CONCLUSIONS**

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The solid model of mechanical structure of the Quill shaft wasinserted in this work. The modal analysis using FEM with SolidWorkssimulation is offered. The three types of critical frequencies Table 2: Results of natural frequencies of the first five mode shapes

| nite mode emper  |             |                              |           |  |
|--|-------------|------------------------------|-----------|--|
| Operating speed of Quill shaft= 50 Hz (3000 rpm)   |             |                              |           |  |
| Mode   | Natural     | Natural Frequencies, Hz(rpm) |           |  |
|  | Bending     | Axial                        | Torsional |  |
| 1.4  | 277.48      | 1081.3                       | 654.8     |  |
| 1st  | (16648.8)   | (64878)                      | (39288)   |  |
| 2nd  | 656.27      | 2653.3                       | 1533      |  |
| 2110   | (39376.2)   | (159198)                     | (91980)   |  |
| 3rd  | 1231.4      | 4118.5                       | 2186.6    |  |
| 510  | (73884)     | (247110)                     | (131196)  |  |
| 4th  | 2019.7      | 6262.9                       | 3532.3    |  |
|  | (121182)    | (375774)                     | (211938)  |  |
| 5th  | 2681.3      | 7930.9                       | 4592.7    |  |
| Stu  | (160878)    | (475854)                     | (275562)  |  |
| 9000   |             |                              |           |  |
| 8000<br>9 7000   | - Axial     |                              |           |  |
| E 6000   | -Bending    |                              | ×         |  |
| ີຊີ 5000   | - Torsional |                              |           |  |
| 토 4000   | -           | -                            |           |  |
| Natural Frequency (Hertz)<br>Natural Frequency (Hertz)<br>0000 0000<br>0000 0000<br>0000<br>0000<br>0000<br>0000<br>0000<br>00000<br>0000 |             |                              |           |  |
| 2 2000<br>2 1000   |             |                              | -         |  |
| 0  |             |                              |           |  |
|  | 1st 2nd     | d 3rd                        | 4th Sth   |  |

Fig. 12 Natural frequencies in (Hz)



Mode Shar

Fig. 13 Natural frequencies in (rpm)

(bending, axial, and torsional) and the inherent mode shapes of the Quill shaft are obtained.

Through the analyses mentioned above, the following conclusions have been deduced:

- 1) All types of natural frequencies are relatively high, i.e., out of range of rotating speed shaft(50 Hz), Table 2.
- The Quill shaft is safe from the standpoint of modal analysis for all three types of vibrations (bending, axial and torsional),since the minimum value among these three types is

almost six times the shaft operating speed, as shown in Table 4 for bending mode.

- 3) Forcing frequency of Quill shaft is 50 Hz, which is very lower than of any of each three types of critical speeds. Therefore, the resonant condition cannot be happened and Quill shaft is not running at critical speed.
- 4) However, practically, if the generator-unit is exposed suddenly to some types of severe loading conditions, such as open circuit or unloading fault, then the speed of Quill shaft will be increasing drastically to values much more than the operating speed and may be exceeded the resonant value. In this case, the redesign of Quill shaft should be carried out.

| Operating speed of Quill shaft=3000 rpm (50 Hz) |                                      |  |
|---|--------------------------------------|--|
| Mode  | Torsional natural<br>frequency (rpm) | Safety margin<br>(=Torsional natural<br>frequency/operating speed) |
| 1st   | 39288                                | 13.1   |
| 2nd   | 91980                                | 30.7   |
| 3rd   | 131196                               | 43.73  |
| 4th   | 211938                               | 70.65  |
| 5th   | 275562                               | 91.85  |

Table 6: Safety margins of torsional mode

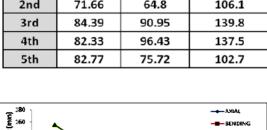
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| URES IN SOLIDWORKS |                             |       |           |  |
|--------------------|-----------------------------|-------|-----------|--|
| RE                 | RESULTANT DISPLACEMENT (mm) |       |           |  |
|                    | MAX VALUES                  |       |           |  |
| Mode               | BENDING                     | AXIAL | TORSIONAL |  |
| 1st                | 85.66                       | 88.71 | 155.6     |  |
| 2nd                | 71.66                       | 64.8  | 106.1     |  |
| 3rd                | 84.39                       | 90.95 | 139.8     |  |
| 4th                | 82.33                       | 96.43 | 137.5     |  |
| 5th                | 82.77                       | 75.72 | 102.7     |  |

Table 3: Maximum resultant displacements for

all modes



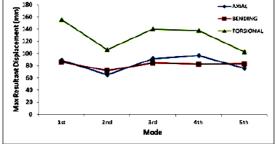


Fig. 14 Relation between maximum resultant displacements for all modes

| Operating speed of Quill shaft=3000 rpm (50 Hz) |                                    |  |
|---|------------------------------------|--|
| Mode  | Bending natural<br>frequency (rpm) | Safety margin<br>(=Bending natural<br>frequency/operating speed) |
| lst   | 16649                              | 5.55   |
| 2nd   | 39376                              | 13.13  |
| 3rd   | 73884                              | 24.63  |
| 4th   | 121182                             | 40.39  |
| 5th   | 160878                             | 53.63  |

Table 5: Safety margins of axial mode

| Operating speed of Quill shaft=3000 rpm (50 Hz) |                                  |  |
|---|----------------------------------|--|
| Mode  | Axial natural<br>frequency (rpm) | Safety margin<br>(=Axial natural<br>frequency/operating speed) |
| 1st   | 64878                            | 21.63  |
| 2nd   | 159198                           | 53.1   |
| 3rd   | 247110                           | 82.4   |
| 4th   | 375774                           | 125.3  |
| 5th   | 475854                           | 158.62   |



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# تحليل السرع الحرجة واشكال الطور لعمود الدوران الملتف التابع لوحدة التوليد التوربينية باستخدام برنامج SolidWorks

## د. فارس قاسم يحيى

كلية الهندسة - قسم هندسة الميكاترونكس- جامعة الموصل

الخلاصة

التوربينية التابعة لمحطة الموصل الغازية باستخدام نسخة تجريبية لبرنامج الحرجة (النبنبات الطبيعية) واشكال الطور لعمود الدوران الملتف لوحدة التوليد التوربينية التابعة لمحطة الموصل الغازية باستخدام نسخة تجريبية لبرنامج SolidWorks بالا عتماد على طريقة العناصر المحددة. ان الوظيفة الاساسية لعمود الدوران الملتف هي حماية وحدة التوليد التوربينية من تاثيرات الاحمال العالية او المفرطة والتي تنتج من الاعطال الكهربائية المتكررة في الشبكة، وذلك بسبب امتلاكه مرونة عالية تجعله قابل لامتصاص القيم العالية اللشوهات المحورية اوالا نحنائية او الالتوائية الناتجة من ظاهرة الرنين والتي تعني تطابق قيم السرع الحرجة مع القيمة التشغيلية لعمود الدوران. يعتبر هذا النوع من التحليل ضروري ومهم لدراسة تاثير الاحمال الانتيالية التي يتعرض لها عمود الدوران الملتف بسبب ظروف الاشتغليلية لعمود الدوران. يعتبر هذا النوع من التحليل ضروري ومهم لدراسة تاثير الاحمال الانتيالية التي يتعرض لها عمود الدوران الملتف بسبب ظروف الاشتغليلية لعمود الدوران. يعتبر هذا النوع من التحليل ضروري ومهم لدراسة تاثير الاحمال الانتقالية التي يتعرض لها الطبيعية) واشكال الطور المطابقة لها بانواعها الثلاثة. تم ايضا عزل وتشخيص ومن ثم حساب وتدوين كل نوع على حدة وذلك بتطبيق لنواع خاصة من القيود المتاحة في البرنامج. لوحظ بشكل واضح ان القيم الاولية او الابتدائية للسرع الحرجة ولكل نوع ذات قيم علية نسبيا ولكر بعدة مرات من قيمة السرعة المتاحة في البرنامج. لوحظ بشكل واضح ان القيم الاولية او الابتدائية للسرع الحرجة ولكل نوع ذات قيم علية نسبيا ولكر بعدة مرات من قيمة على هذا النوع من اعمدة الدوران الملتف بحيث تضمن عدم حدوث ظاهرة الرنين في اي مرحلة من مراحل التشغيل. استنتج من جميع التحليلات التي اجريت على هذا النوع من اعمدة الدوران الملتف بحيث تضمن عدم حدوث ظاهرة الرنين في من هذه الناحية، وان ظاهرة الرنين المو على هذا النوع من اعمدة الدوران والتي تحمين التي ينجم عنها مشاكل تصميمية متعددة مثل التشوء الدائمي او الرسين التي تعني تطابق السرعة على هذا النوع من اعمدة الدوران والتي التي لي المي المنيك تصميمية متعددة مثل التشوه الدائمي او الاليو الو البرة ال حمالية السرعة الحرجة لعمود الدور ان مو سر عالائتي ماتي الا هتزازات المذكورة، بعيد جد الذمن من هذه الناحية الول المرا البحش ال الحرجة لعمود الدور

